# Evolvability in Grammatical Evolution

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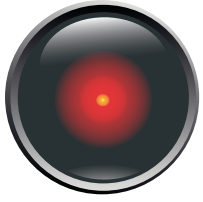

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GECCO, 17/7/2017, Berlin (Germany)

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# Grammar-based GP

- grammar-based GP approaches are popular
  - user just needs to provide the grammar and the fitness function
  - ensure valid individuals

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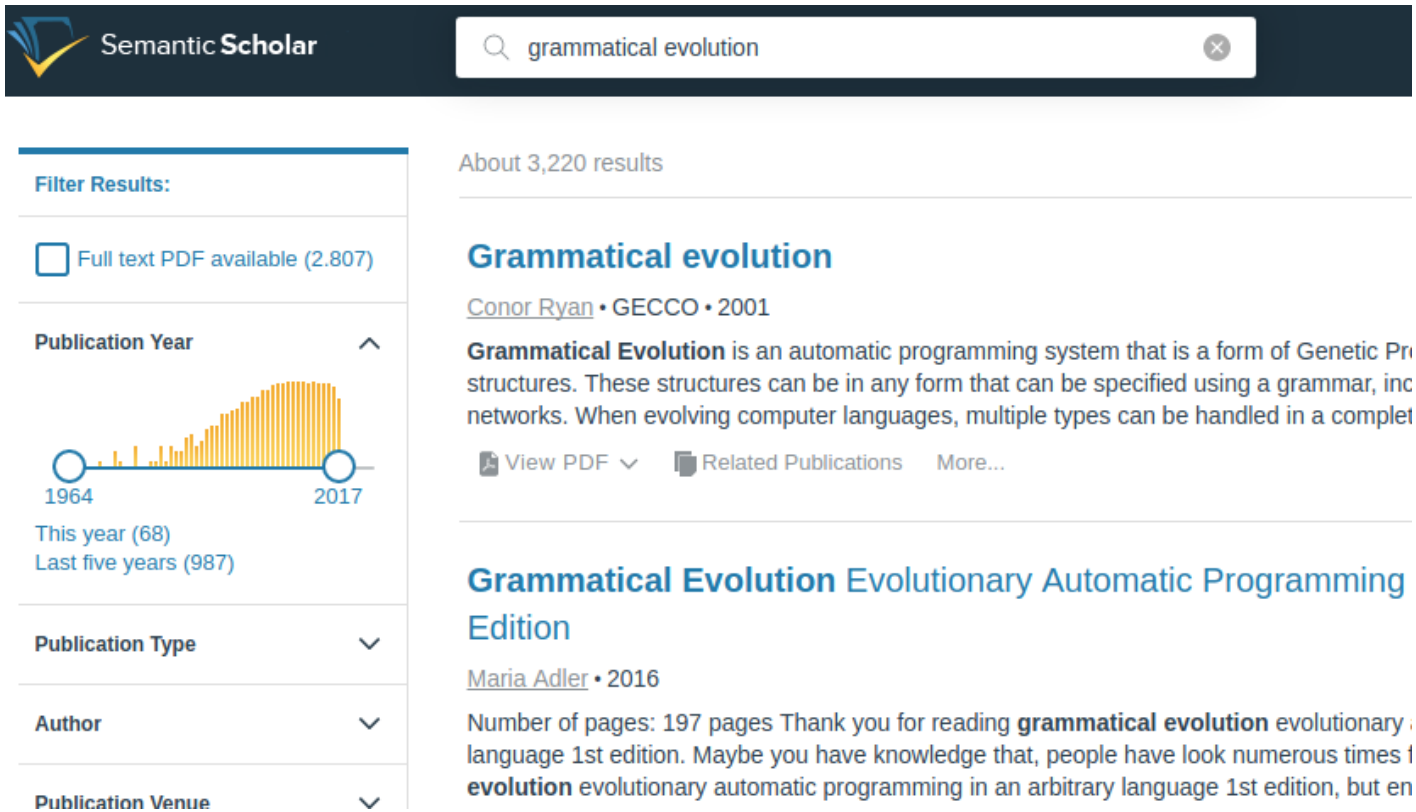
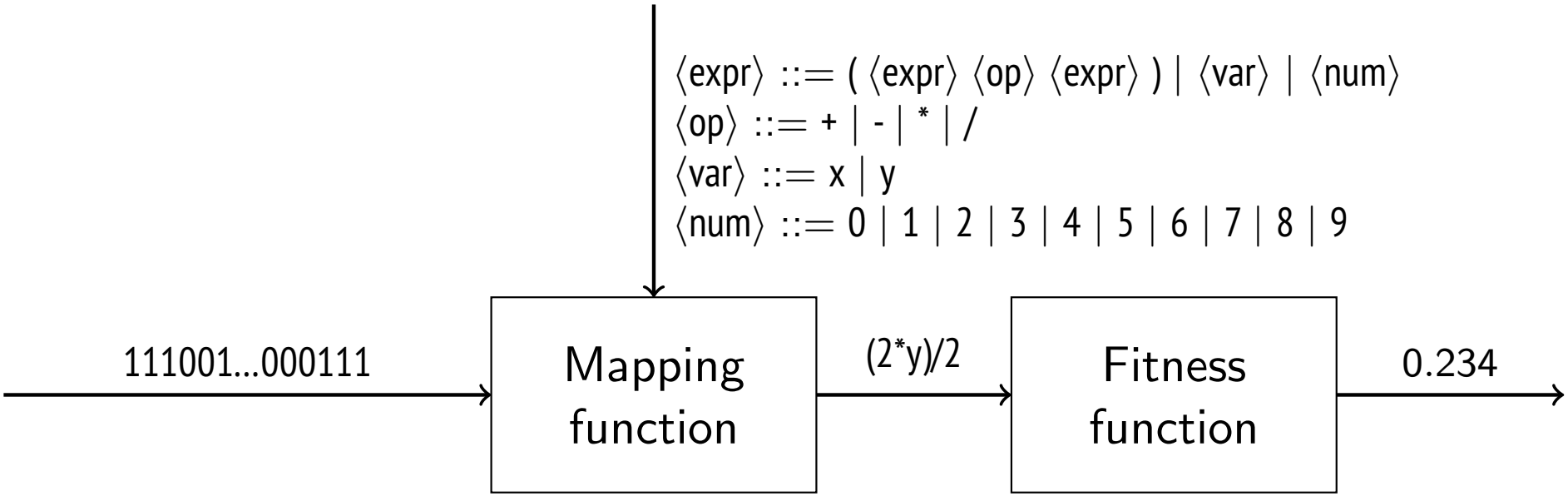
  - user just needs to provide the grammar and the fitness function
  - ensure valid individuals
- among them, Grammatical Evolution (GE) widely used



# GE

Three representations for the individual:

- genotype: a bit string
- phenotype: a string of the language defined by the CFG
- fitness



# Genotype-phenotype mapping

Salient trait:

- allows using (standard) bit string genetic operators
- indirect mapping, poor variational intheritance
  - many variants proposed (BGE, $\pi$GE, SGE, ...)
  - large/hot debate: "GE mapping is bad!"[1], "No! It's good!"[2][3]

[1] Whigham, Dick, and Maclaurin, "On the mapping of genotype to phenotype in evolutionary algorithms".

[2] Squillero and Tonda, "(Over-)Realism in evolutionary computation: Commentary on "On the mapping of genotype to phenotype in evolutionary algorithms" by Peter A. Whigham, Grant Dick, and James Maclaurin".

[3] Ryan, "A rebuttal to Whigham, Dick, and Maclaurin by one of the inventors of grammatical evolution: Commentary on "On the mapping of genotype to phenotype in evolutionary algorithms" by Peter A. Whigham, Grant Dick, and James Maclaurin".

# Mapping and properties

Mapping:

- invalidity: how often the mapping "fails"?
- redundancy[4]: is part of the genotype not used?
- degeneracy[4]: how often different genotypes are mapped to the same phenotype?

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Mapping + genetic operators + fitness:

- evolvability: is the child fitter than its parents?

Many studies on locality/degeneracy, none on evolvability $\Rightarrow$ our paper!

[4] Often, redundancy is used for degeneracy

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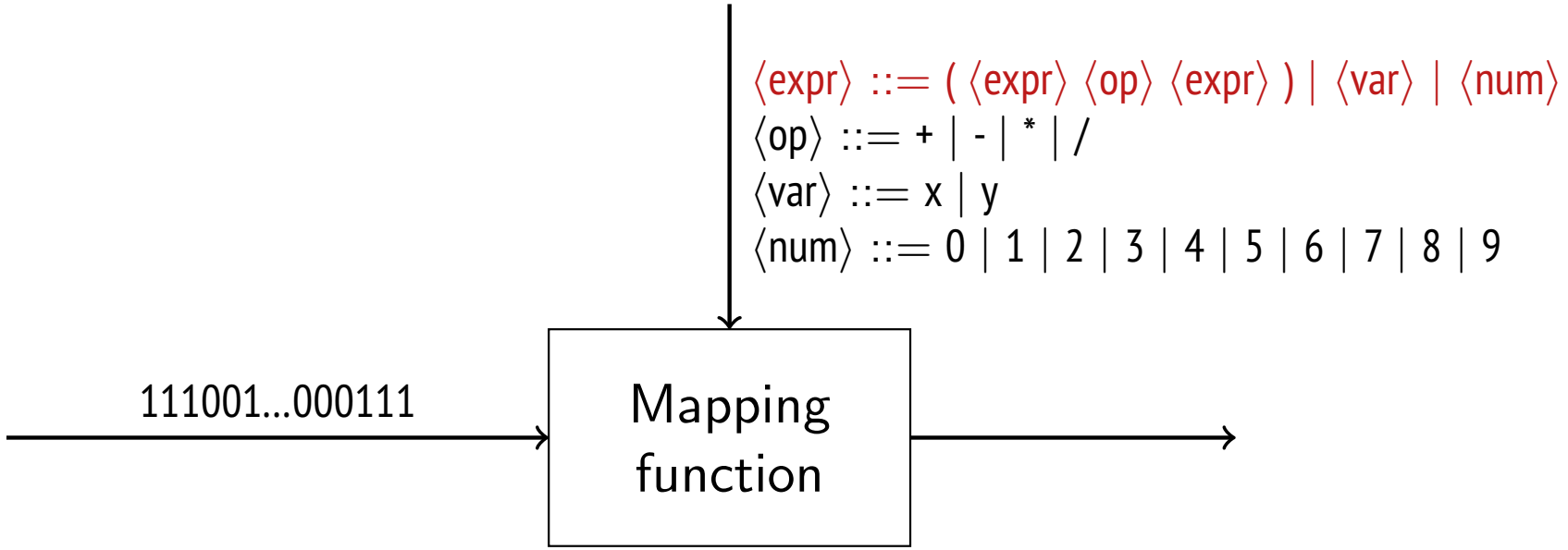
# GE mapping in brief

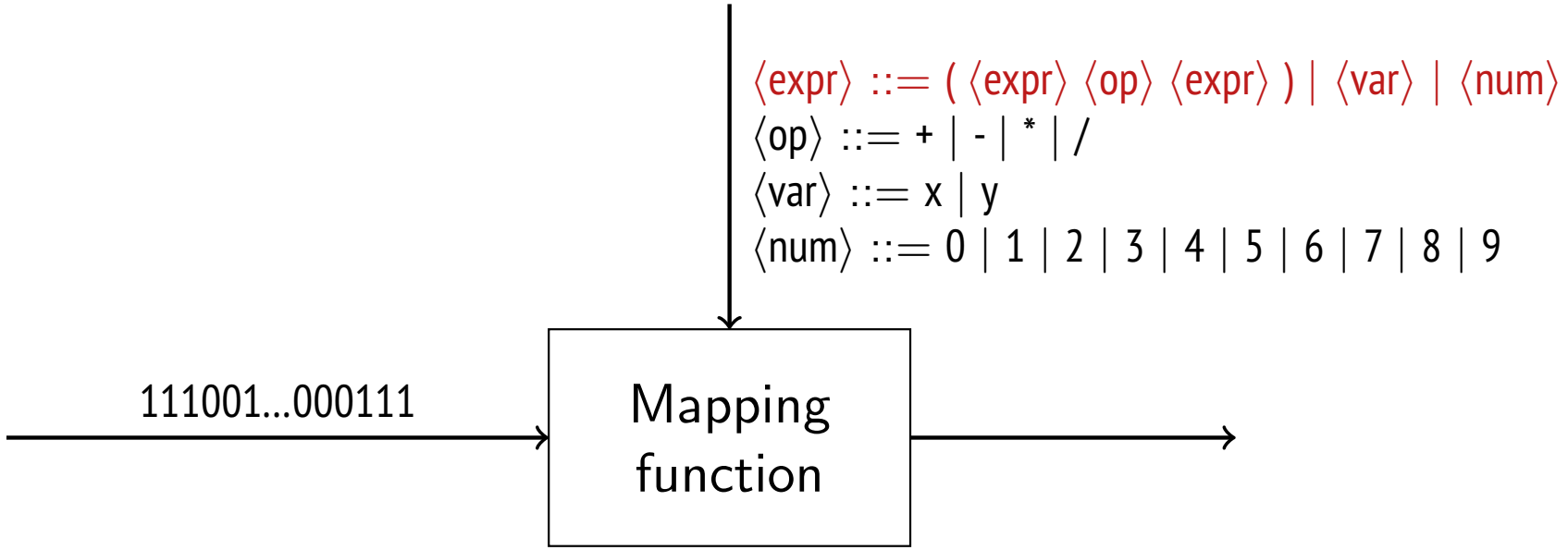



| | | | | | | |
|---|---|---|---|---|---|---|
| Genotype (bits) | 11100111 | 11110000 | 10100001 | 01110001 | 01001101 | 00000111 |
| Genotype (ints) | 231 | 15 | 133 | 142 | 178 | 224 |

i=0
w=0
p=⟨**expr**⟩

$231 \mod 3 = 0$

# GE mapping in brief

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i=1

w=0

p= ( ⟨**expr**⟩ ⟨op⟩ ⟨expr⟩ )

$15 \mod 3 = 0$

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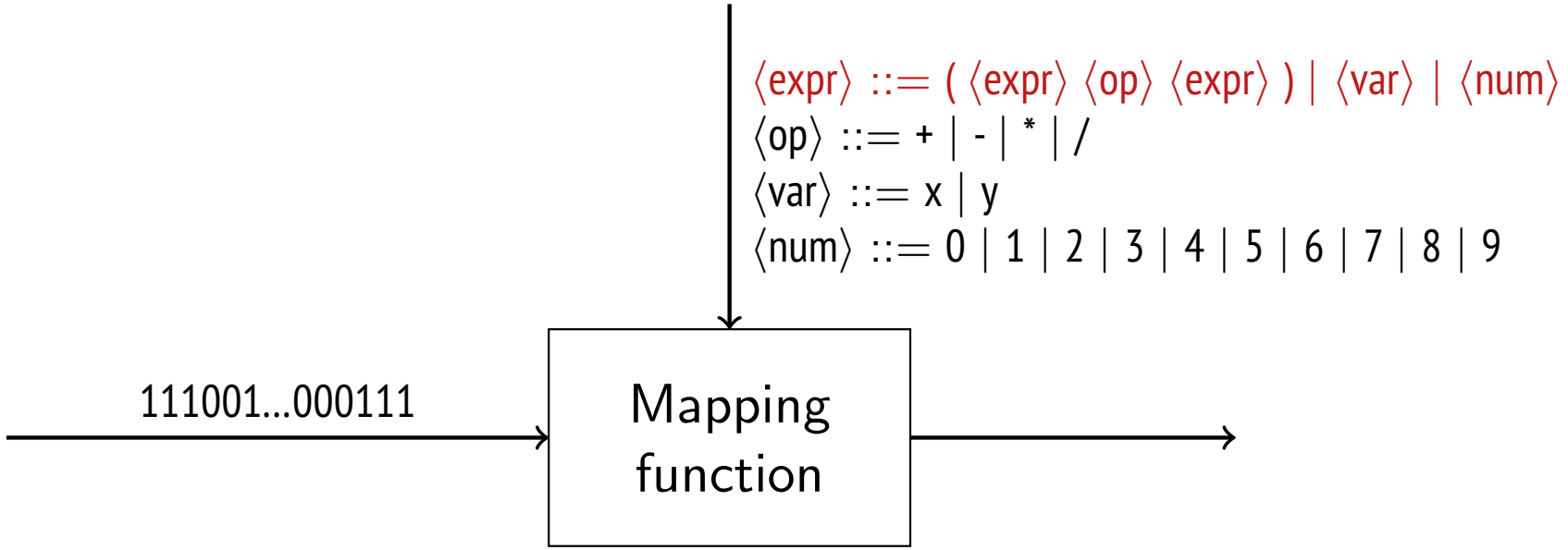


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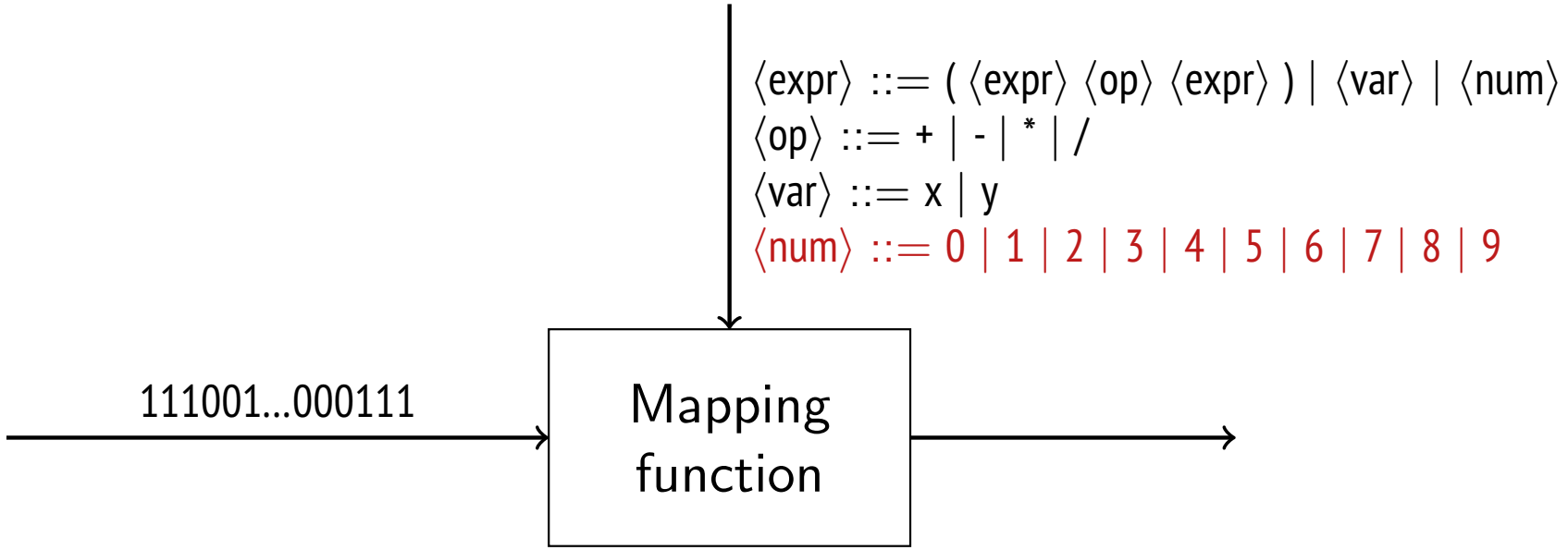
i=2

w=0

p= ( ( ⟨**expr**⟩ ⟨op⟩ ⟨expr⟩ ) ⟨op⟩ ⟨expr⟩ )

$133 \mod 3 = 1$

# GE mapping in brief

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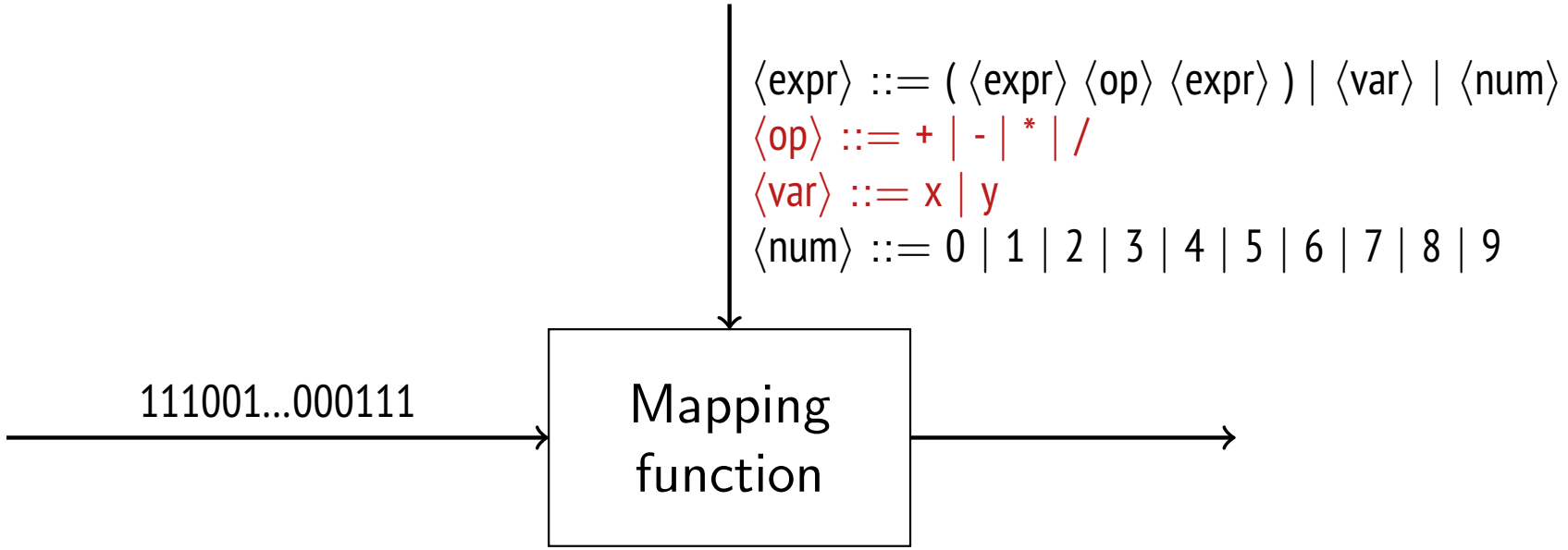
i=3
w=0
p= ( ( ⟨**expr**⟩ ⟨op⟩ ⟨expr⟩ ) ⟨op⟩ ⟨expr⟩ )

$142 \mod 10 = 2$

# GE mapping in brief

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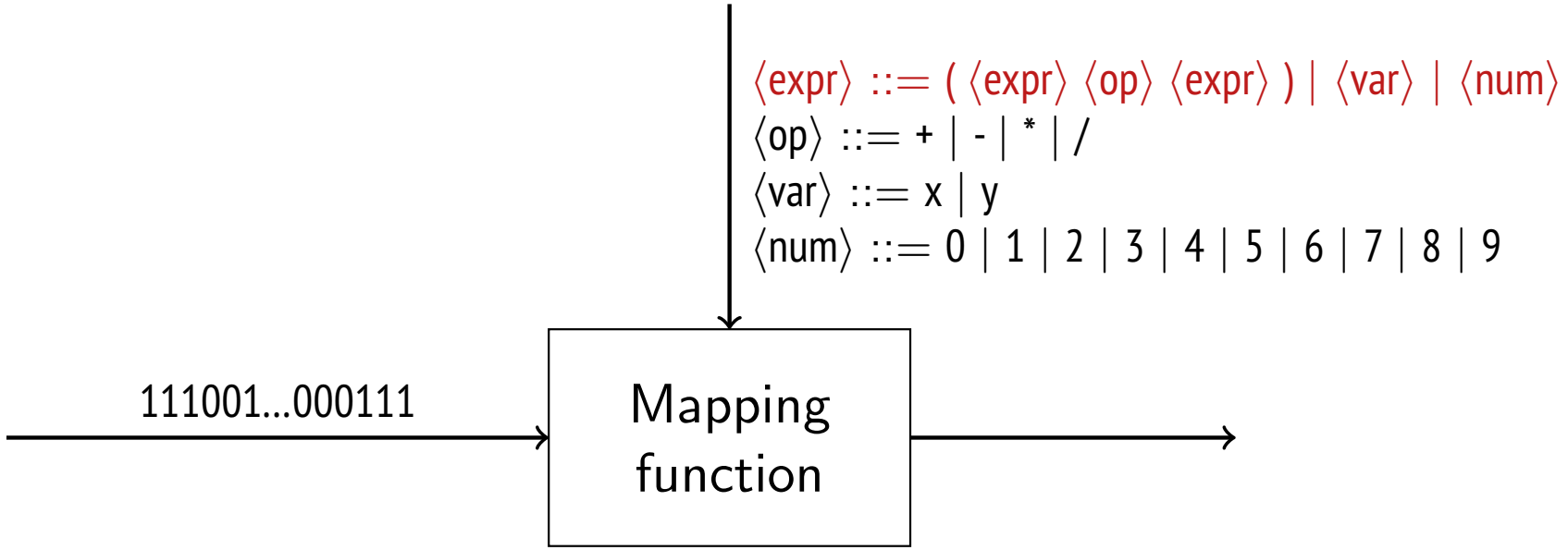
i=4

w=0

p= ( ( 2 ⟨**op**⟩ ⟨expr⟩ ) ⟨op⟩ ⟨expr⟩ )

$178 \mod 4 = 2$

# GE mapping in brief

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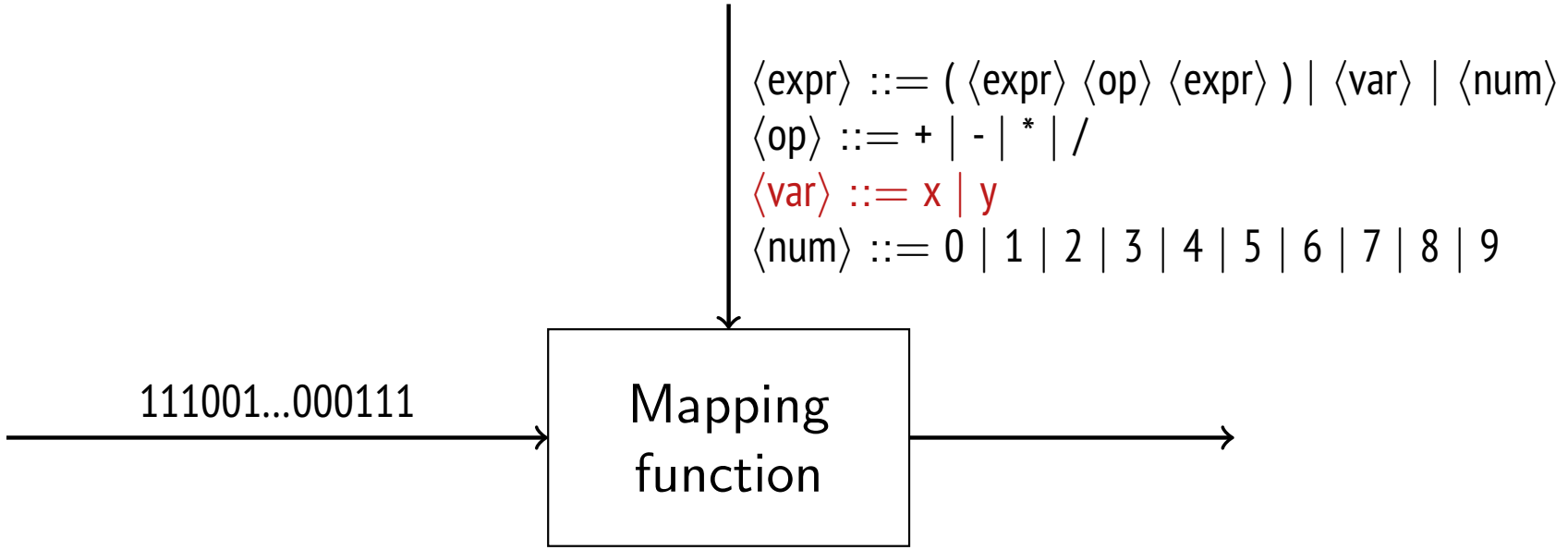
i=5
w=0
p= ( ( 2 * ⟨**expr**⟩ ) ⟨op⟩ ⟨expr⟩ )

$224 \mod 3 = 2$

# GE mapping in brief

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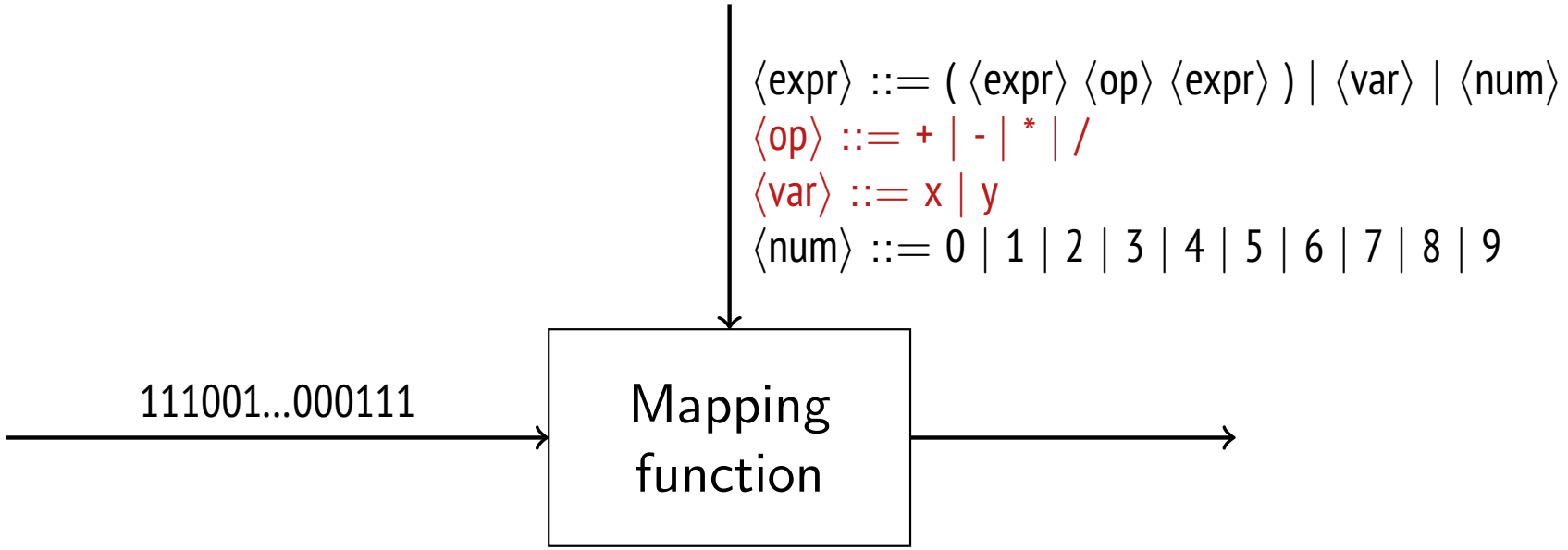
i=0
w=1
p= ( ( 2 * ⟨**var**⟩ ) ⟨op⟩ ⟨expr⟩ )

$231 \mod 2 = 1$

# GE mapping in brief

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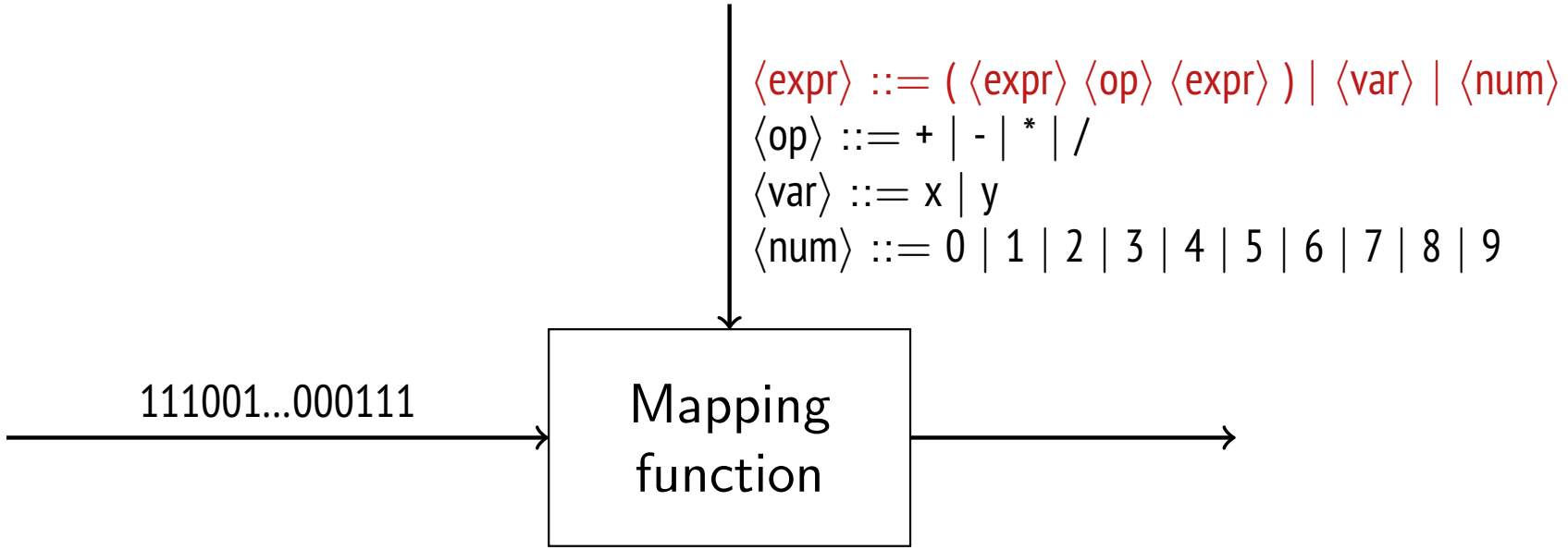
i=1
w=1
p= ( ( 2 * y ) ⟨**op**⟩ ⟨expr⟩ )

$15 \mod 4 = 3$

# GE mapping in brief

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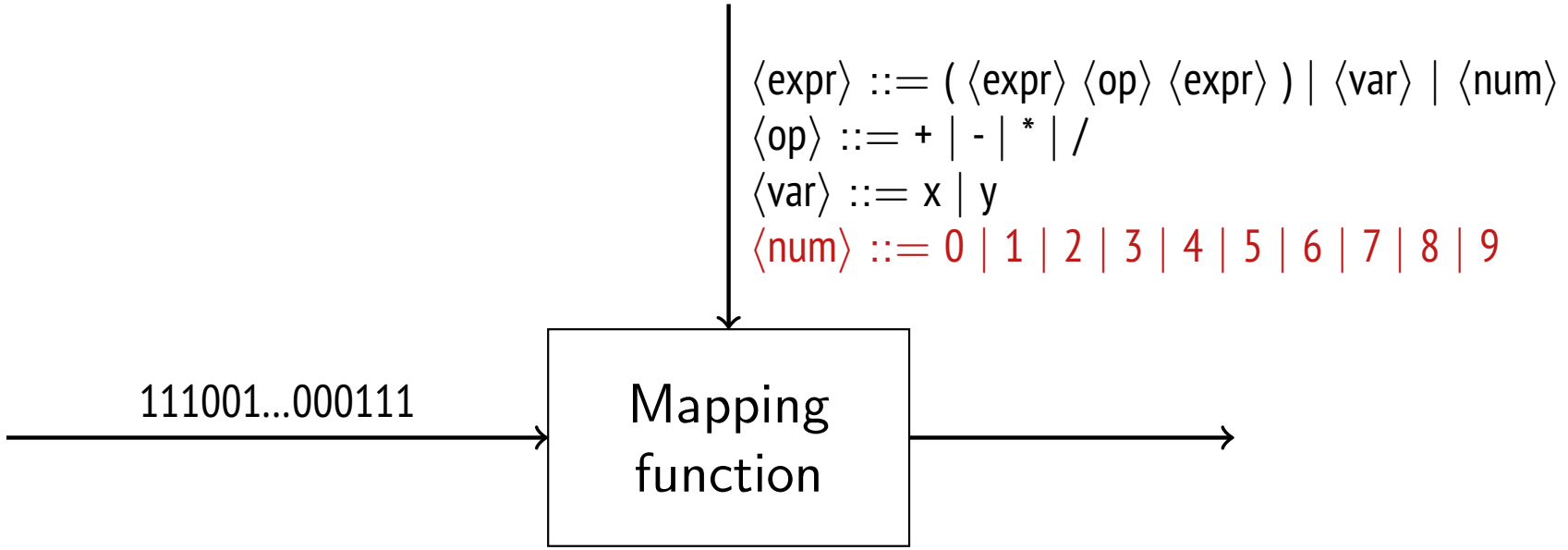
i=2

w=1

p= ( ( 2 * y ) / ⟨**expr**⟩ )

$133 \mod 3 = 1$

# GE mapping in brief

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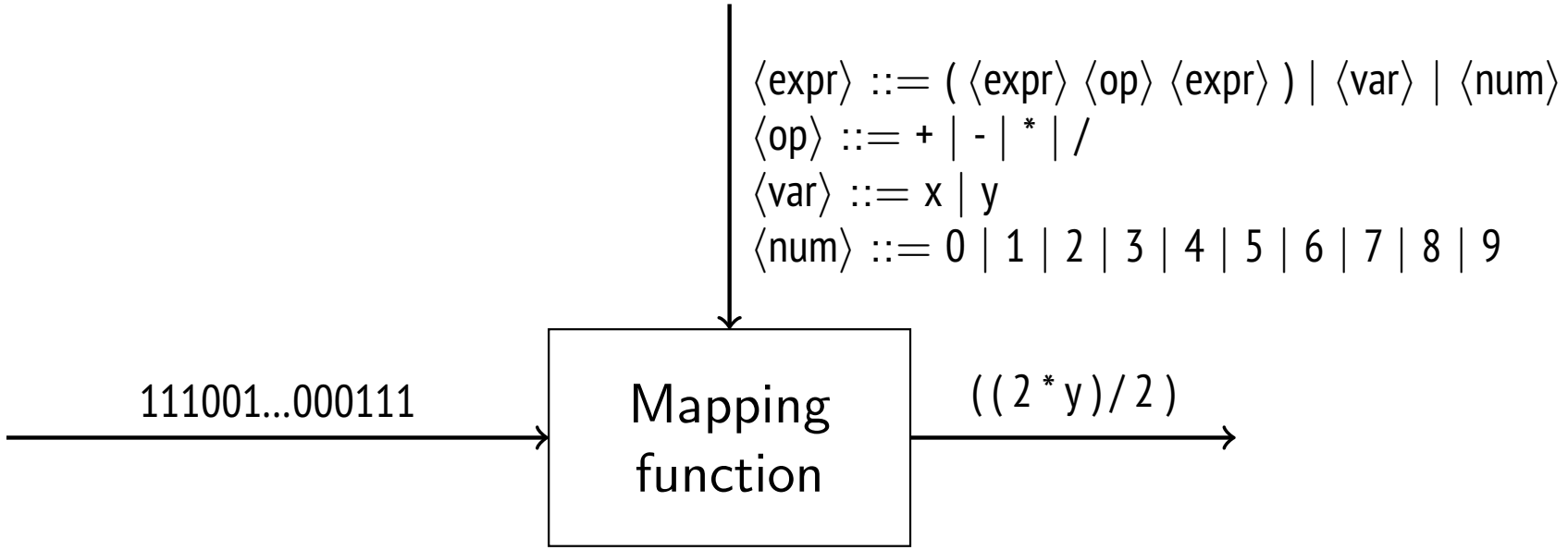
i=3
w=1
p= ( ( 2 * y ) / ⟨**num**⟩ )

$142 \mod 10 = 2$

# GE mapping in brief

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i=3
w=1
p= ( ( 2 * y ) / 2 )

# GE variants

- BGE: breadth first instead of leftmost
- $\pi$GE: non-terminal to expand chosen using the genotype
- SGE: structured genotype (int string), no genotype reuse, no modulo rule, custom operators
  - designed for better locality and lower degeneracy

# Evolvability

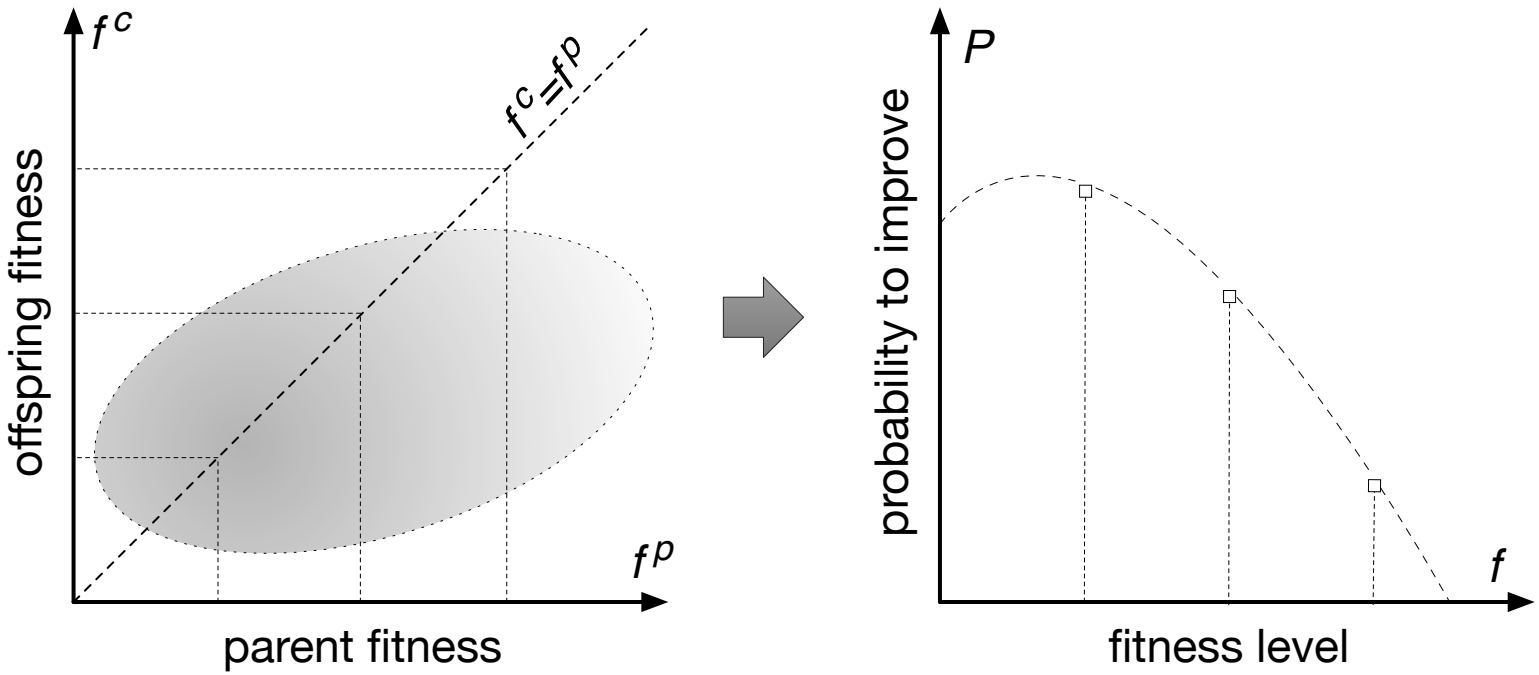
"is the child fitter than its parents?"

- a tool for *fitness landscape analysis*
  - problem hardness from point of view of search heuristic
- chosen metric: fitness cloud[5] $\rightarrow$ **fitness-probability cloud** (FPC)[6] $\rightarrow$ escape probability $\rightarrow$ accumulated escape probability (AEP)

[5] Verel, Collard, and Clergue, "Where are bottlenecks in nk fitness landscapes?"

[6] Lu, Li, and Yao, "Fitness-probability cloud and a measure of problem hardness for evolutionary algorithms".

# Fitness-probability cloud




- AEP: average of escape probability across bins

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# Procedure

Problems and variants:

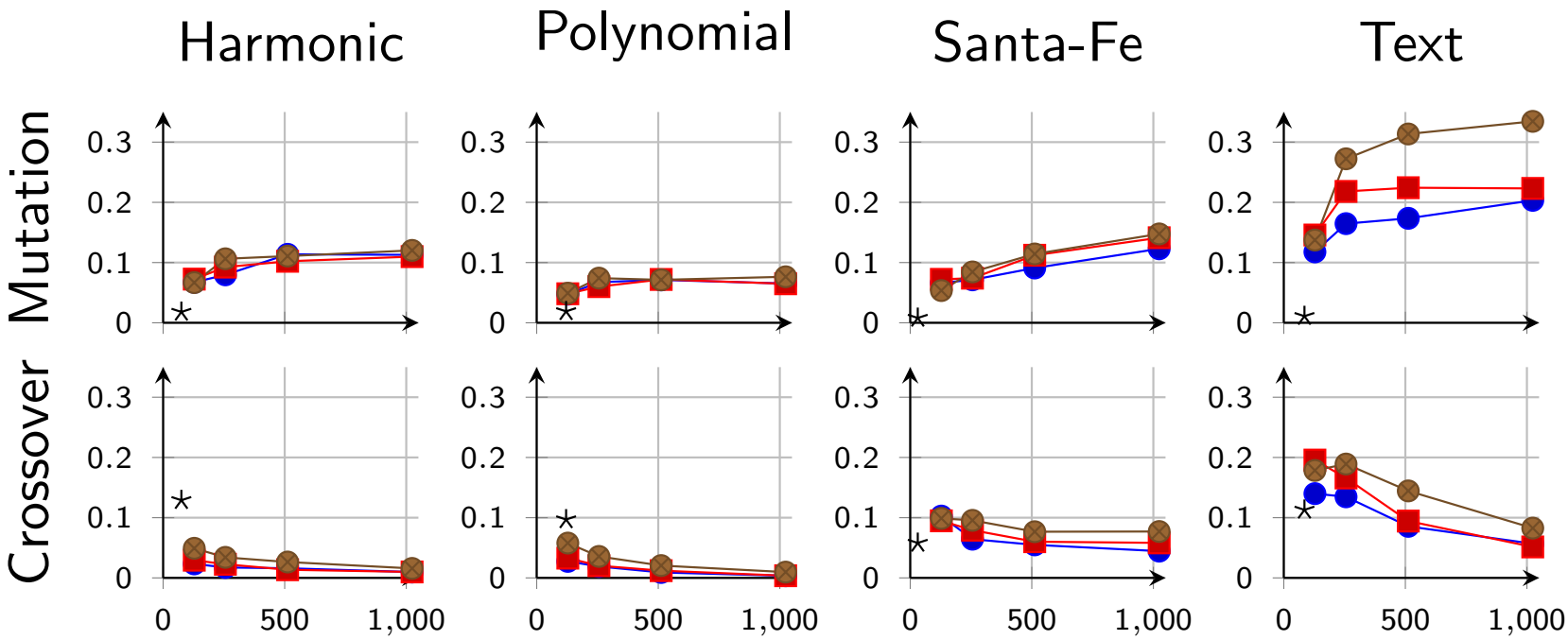
- 4 benchmark problems: harmonic, polynomial, Santa-Fe, text
- GE, BGE, $\pi$GE, SGE
  - 4 genotype sizes: 128, 256, 512, 1024 bit (except SGE)

Procedure:

1. randomly generate 300 (pair of) parents
2. for each, apply 30 times the genetic operator
3. for each $\langle \text{parents}, \text{child} \rangle$, map, compute fitness, compute FPC and AEP
   - one FPC and one AEP value comes from $300 \times 30$ "points"

No dynamical aspects (selection, replacement)!

# Results: AEP vs. genotype size



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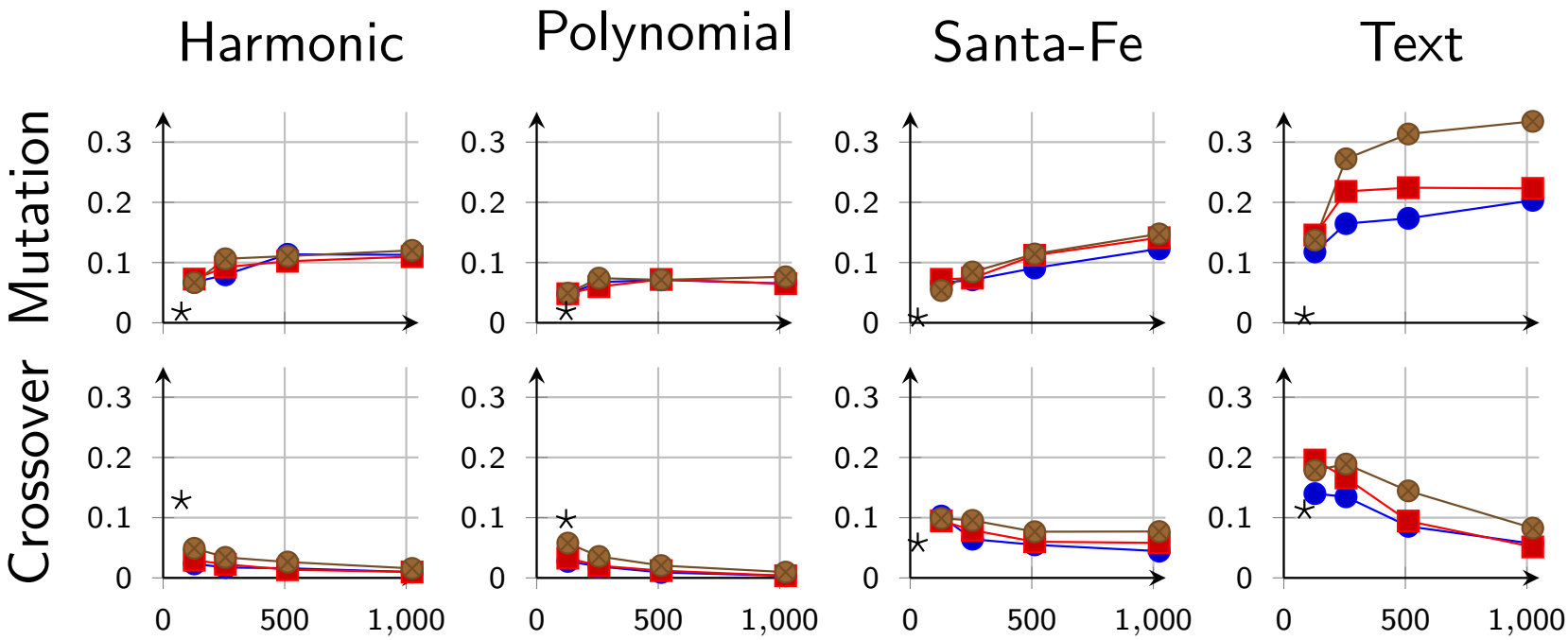


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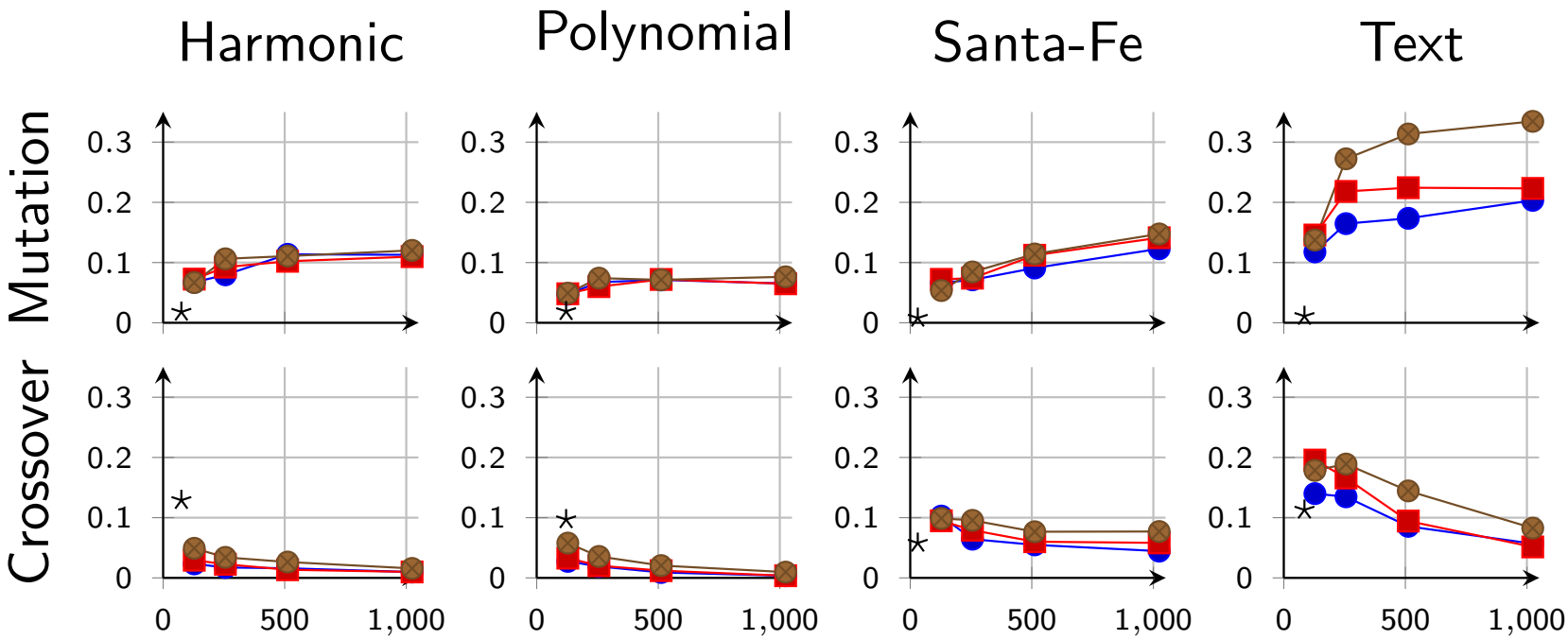


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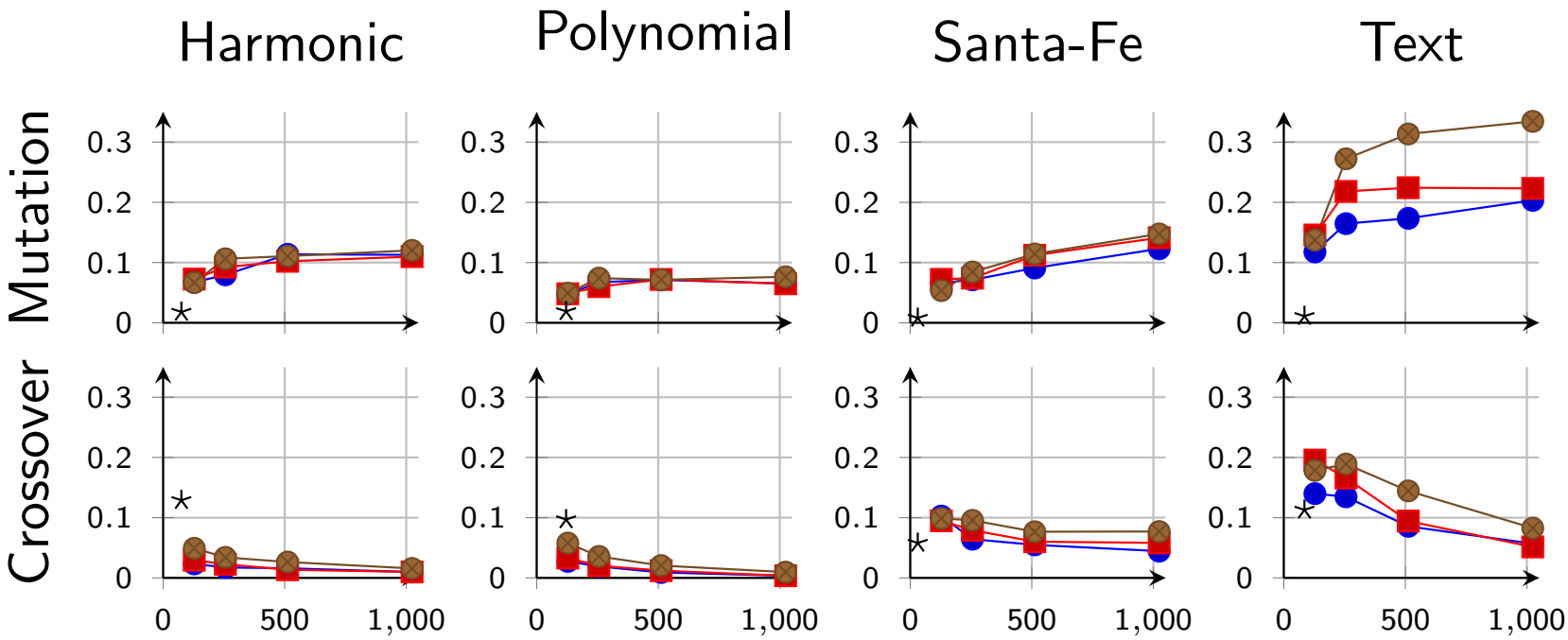


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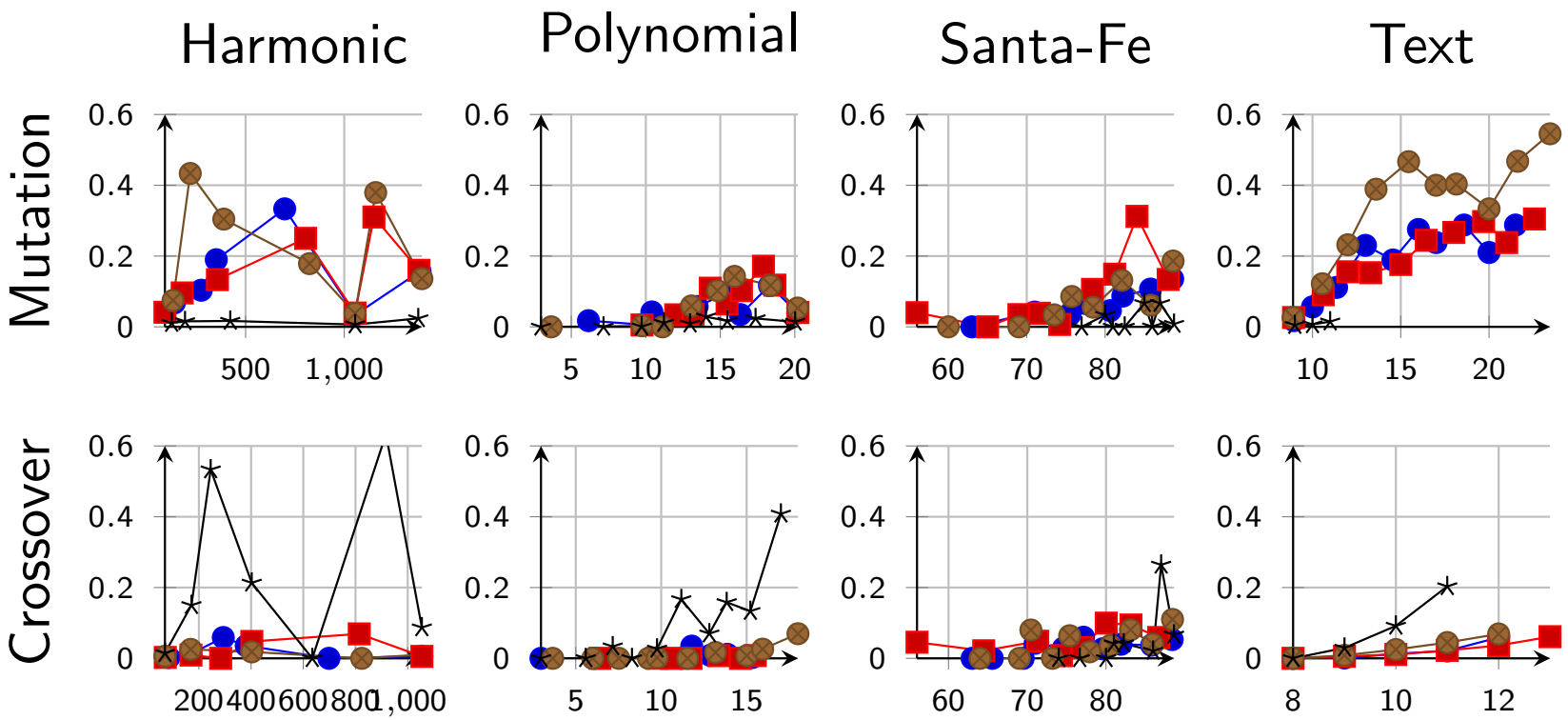
- differences among problems

# Fitness-probability cloud



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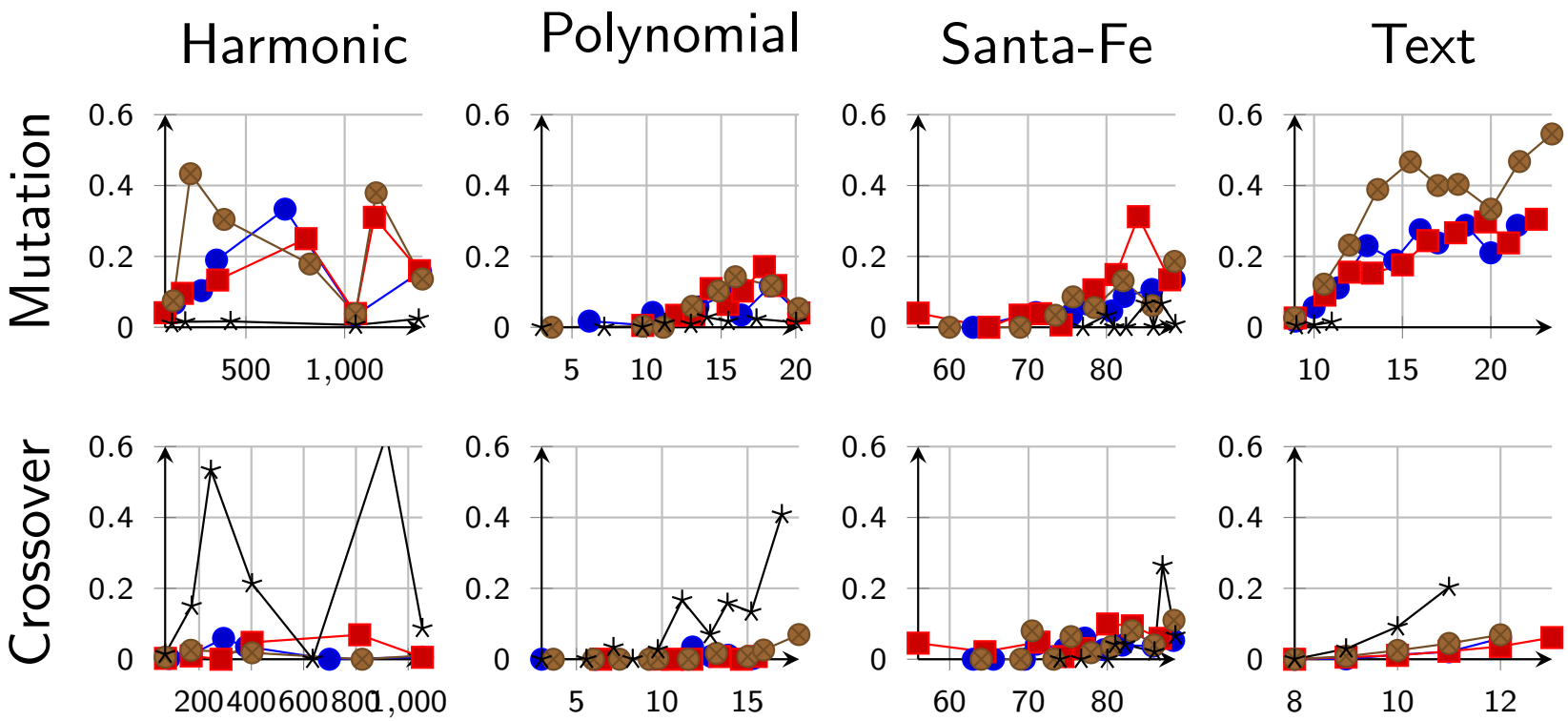


- the better the fitness, the lower the escape probability

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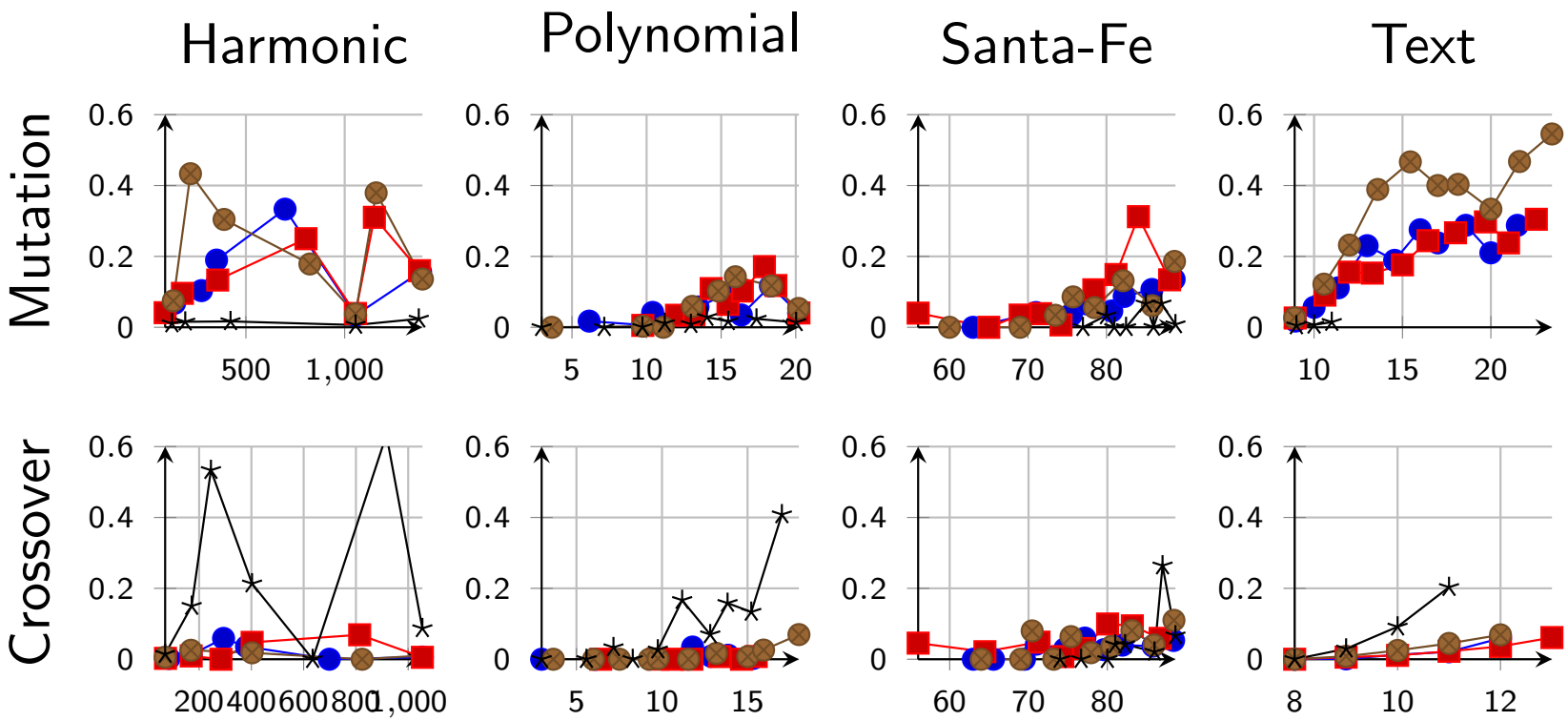


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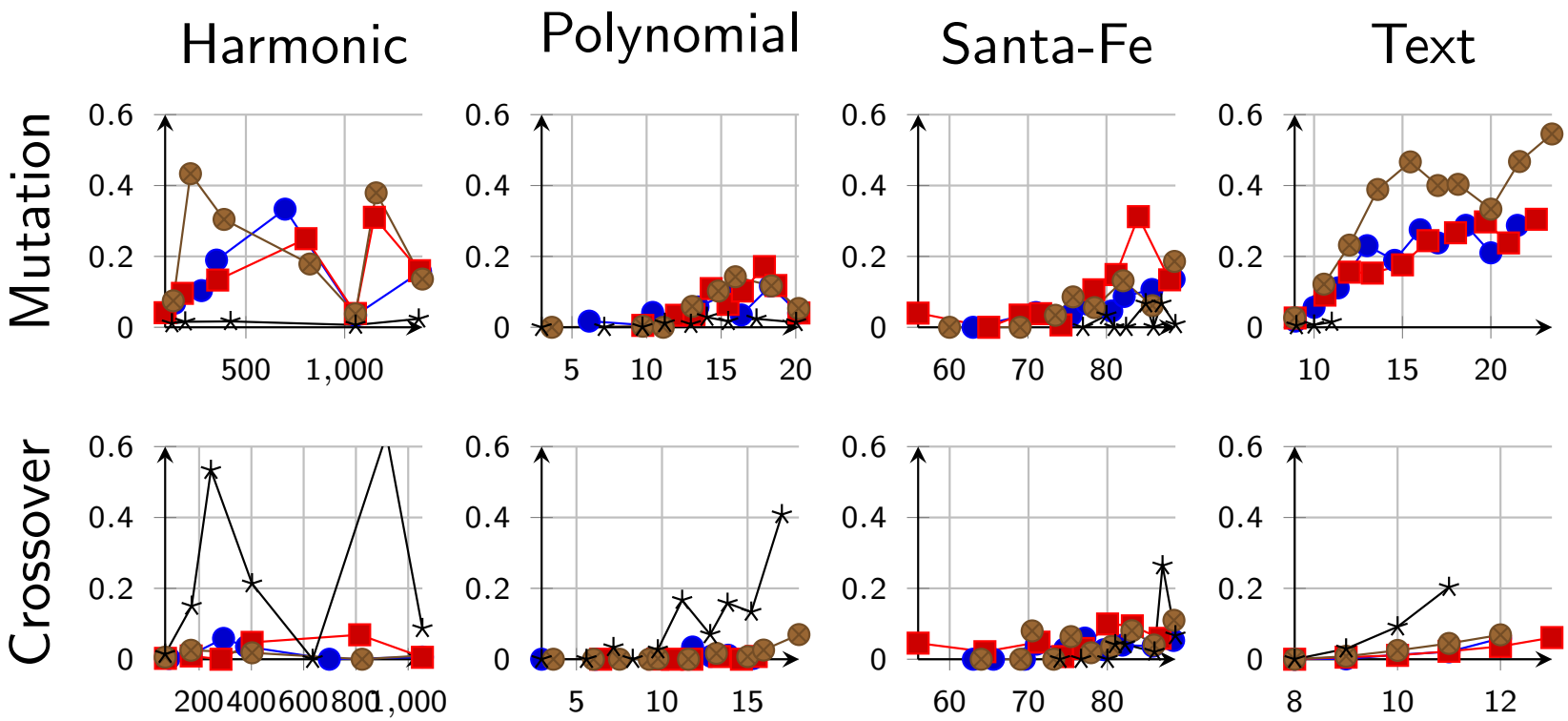


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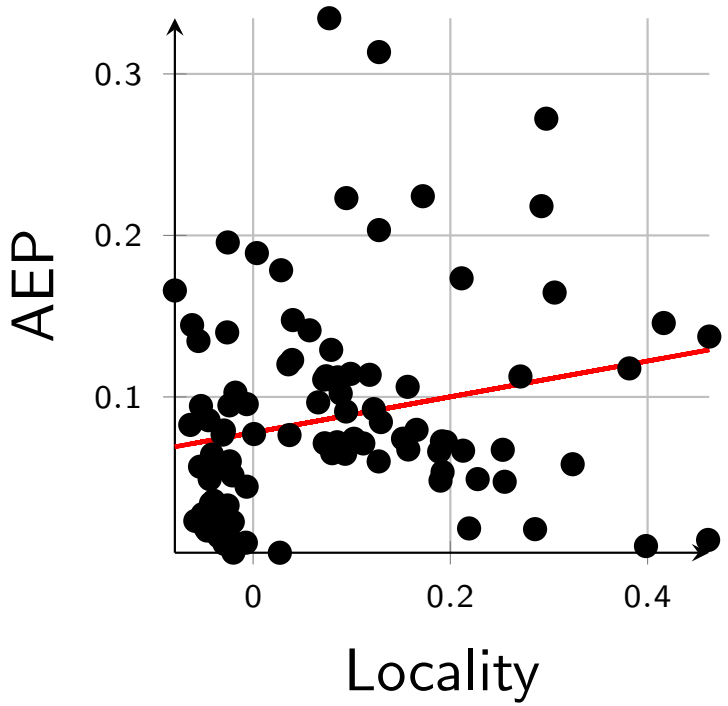
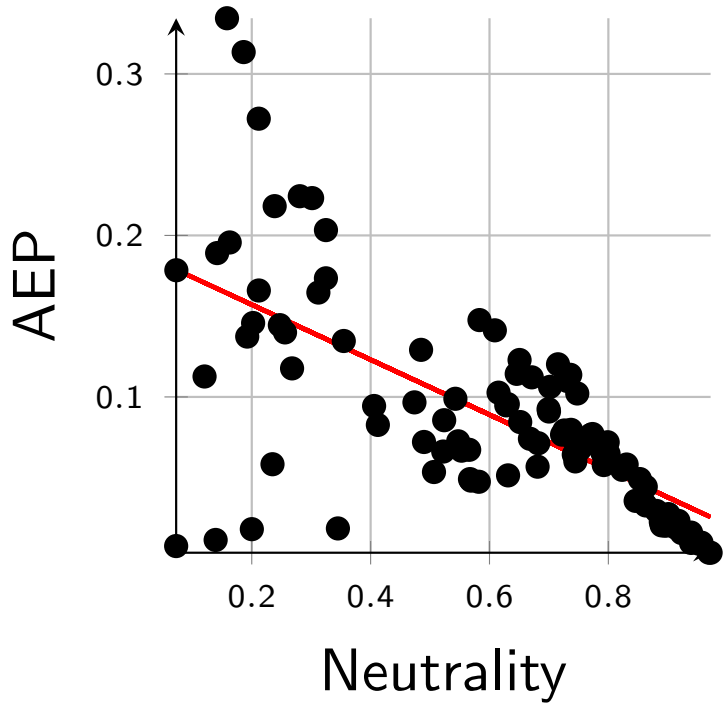
- neutrality
  percentage of cases in which the child genotypes is different from both parent genotypes (that is, $g_p^1 \neq g_c \neq g_p^2$) and the child phenotype is equal to at least one parent phenotype (i.e., $g_c = g_p^1 \vee g_c = g_p^2$)

# Locality/neutrality and evolvability

One point:

- a combination of: variant, problem, genotype size, operator
- coordinates come from $300 \times 30$ numbers



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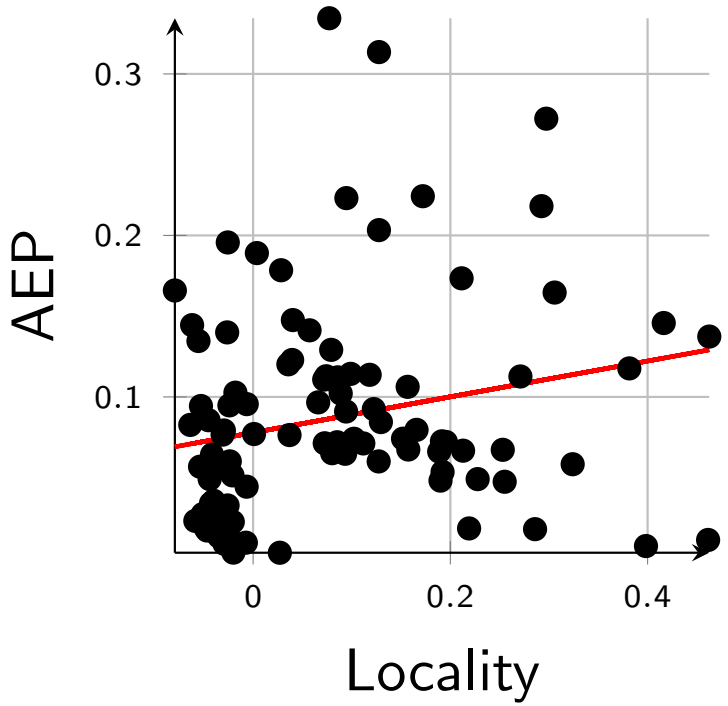


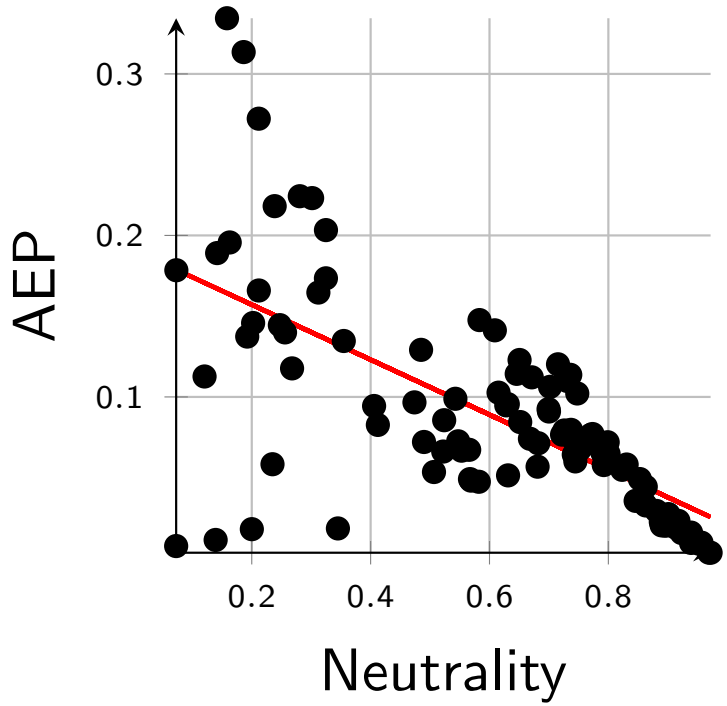


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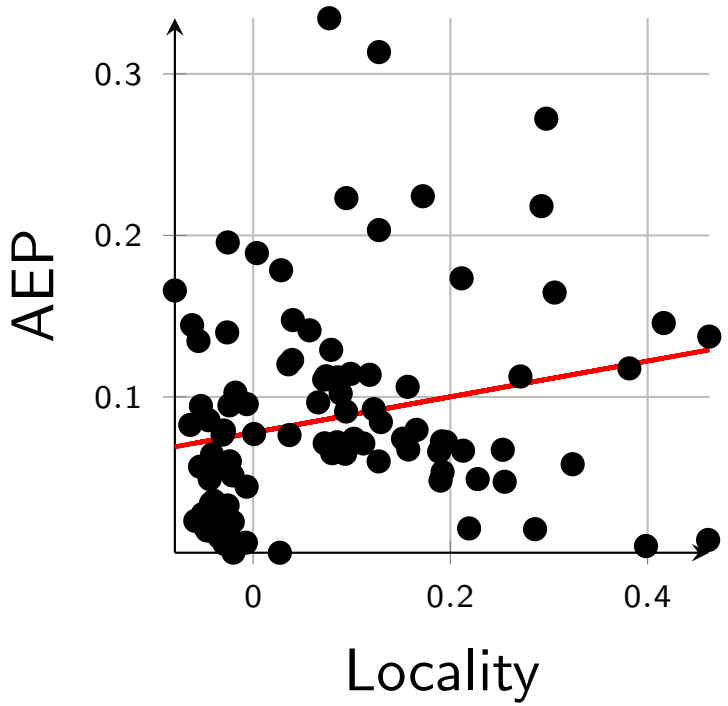


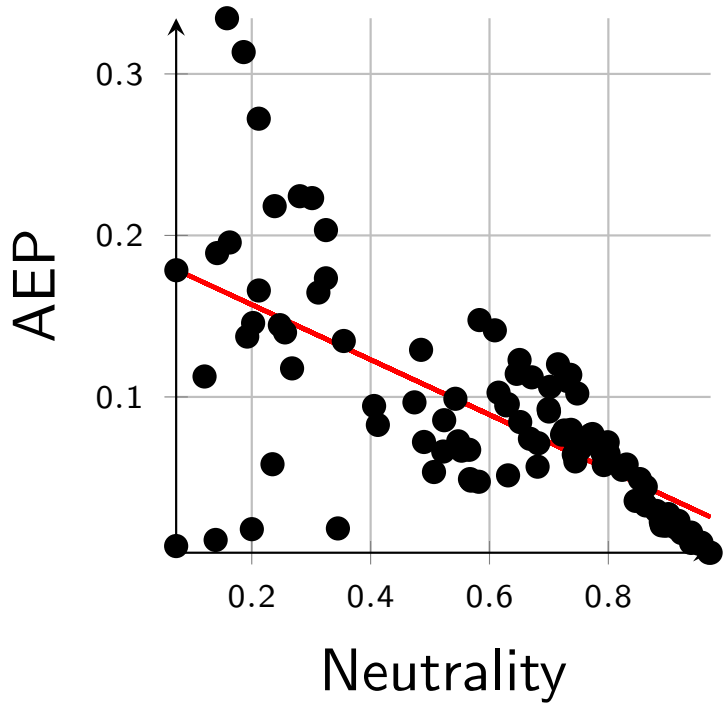


- neutrality more (neg.) correlated than locality
- intuition: no different child, no fitter child

# Conclusions

First study of evolvability in GE:

- none among problem, mapping, genotype size, genetic operator strongly affects evolvability
- locality weakly correlated with evolvability
- neutrality well correlated with evolvability, in particular for higher values

Thanks!