

On the Automatic Design of a Representation for Grammar-based Genetic Programming

Eric Medvet and Alberto Bartoli

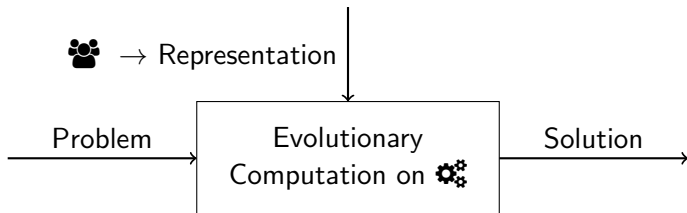
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EuroGP, 5/4/2018, Parma (Italy)

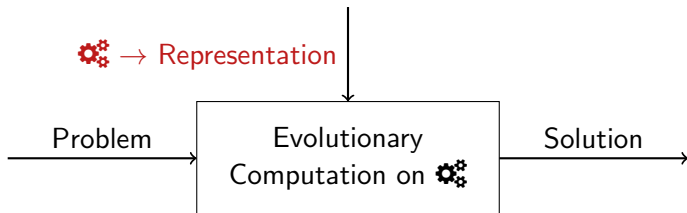
What we know

We can use a machine for obtaining automatically a good solution for “any” problem at hand, by means of an Evolutionary Computation.



What we know

We can use a machine for obtaining automatically a good solution for “any” problem at hand, by means of an Evolutionary Computation.



Can we **obtain automatically** a good representation too?

Table of Contents

- 1 Background and motivation
- 2 Evolving a representation
- 3 Experimental evaluation



Then and now

2007 (30 years perspective): “perhaps the most difficult and least understood area of EA design is that of adapting its internal representation.”¹

¹De Jong, “Parameter setting in EAs: a 30 year perspective”, 2007.

²Spector, “Introduction to the peer commentary special section on “On the Mapping of Genotype to Phenotype in Evolutionary Algorithms” by Peter A. Whigham, Grant Dick, and James Maclaurin”, Sept. 2017.

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2017: “How should the representations that are used in evolutionary algorithms, on which variation and selection act, be **chosen** and **justified**?”²

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2007 (30 years perspective): “perhaps the most difficult and least understood area of EA design is that of adapting its internal representation.”¹

2017: “How should the representations that are used in evolutionary algorithms, on which variation and selection act, be **chosen** and **justified**?”²

Large debate, many arguments/POVs, weak agreement. . .

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Choose/design and justify: so far

- How to choose/design?
- How to justify?



Choose/design and justify: so far

- How to choose/design? → inspiration by Nature/guidelines
- How to justify?



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- How to justify? → study its properties



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Outcome:



Nature not inspiring enough



no wide agreement on practical *guidelines*

Choose/design and justify: so far

- How to choose/design? → inspiration by Nature/guidelines
- How to justify? → study its properties

Outcome:

- ☹ *Nature* not inspiring enough
- ☹ no wide agreement on practical *guidelines*
- ☺ some agreement on which *properties* matter
 - variational inheritance principle

Idea!

Design automatically the representation
aiming at obtaining good properties

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Design automatically the representation
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Sound! Already conjectured and partially attempted:

- “design guidelines [. . .] may be met not through clever engineering [. . .], but through the action of the evolutionary process itself”³
- meta-evolution, self-adaptation, hyper-heuristic

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We did it for a challenging case: GE!

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Why Grammatical Evolution (GE)?

- Great practical interest
- Inspired by Nature
- Challenging, widely studied **indirect representation**



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Why Grammatical Evolution (GE)?

- Great practical interest
 - works on any problem with solutions described by a CFG
 - trendy!
- Inspired by Nature
 - (at least loosely)
- Challenging, widely studied **indirect representation**
 - familiar bit string genotype
 - many experimental studies on properties: redundancy, locality, uniformity
 - many representation variants: GE, π GE, HGE/WHGE (and SGE)



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What we need

Problem: evolving a representation with good properties for GE

We need to:

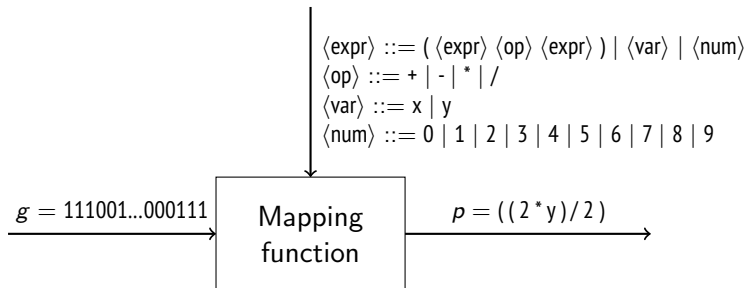
- 1 define the representation (of grammar-based representations)
- 2 define the fitness function



Representation of grammar-based representations

Bit string grammar-based representation, a *mapping function* which:

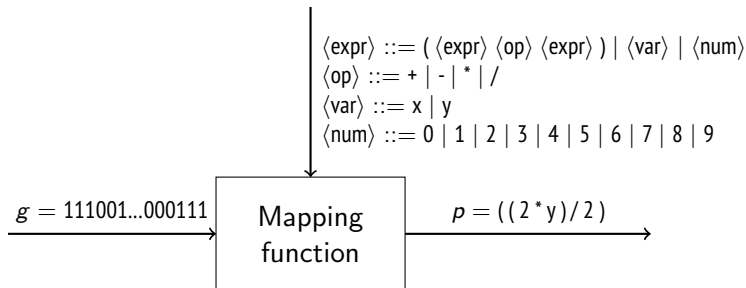
- maps any bit string to a **valid** string w.r.t. the user-provided CFG
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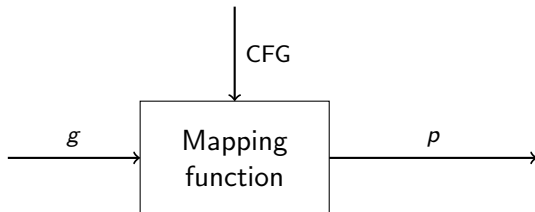
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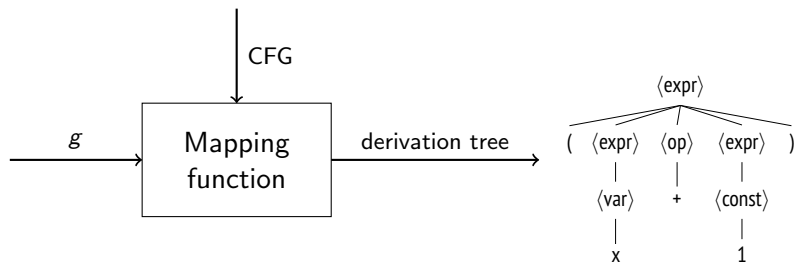
CFGs may be (and usually are) recursive \rightarrow infinite languages!



Mapping function template

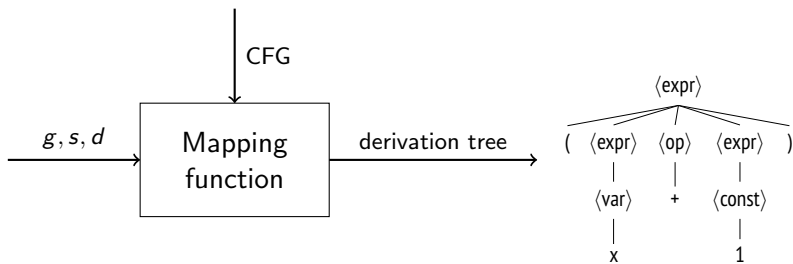


Mapping function template



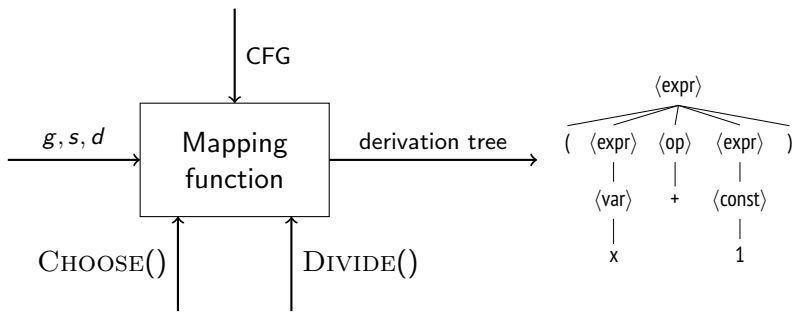
- Returns a derivation tree

Mapping function template



- Returns a derivation tree
- Recursive

Mapping function template



- Returns a derivation tree
- Recursive
- Actual behavior depends on CHOOSE() and DIVIDE()

Mapping function template: details

A recursive $\text{MAP}(g, s, d)$ function (inspired by WHGE):

- 1 consider derivation options for symbol s
- 2 if depth $d > d_{\max}$
 - 1 choose a predefined option
 else
 - 1 choose option with $\text{CHOOSE}(g, \dots)$
- 3 “split” g in pieces with $\text{DIVIDE}(g, \dots)$
- 4 for each piece g_i
 - 1 call $\text{MAP}(g_i, s_i, d + 1)$
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CHOOSE() and DIVIDE()

A language for the 2 functions, as a CFG:

```

<mapper> ::= <n> <lg>
  <n> ::= <const.n> | <var.n> | <fun.n.g> ( <g> ) | <fun.n.n,n> ( <n> , <n> ) | <fun.n.ln> ( <ln> ) |
  <fun.n.ln,n> ( <ln> , <n> ) | <fun.n.lg> ( <lg> )
  <ln> ::= <var.ln> | <fun.ln.n> ( <n> ) | <fun.ln.n,n> ( <n> , <n> ) | apply ( <fun.n.g> , <lg> )
  <g> ::= <var.g> | <fun.g.g,n> ( <g> , <n> ) | <fun.g.lg,n> ( <lg> , <n> )
  <lg> ::= <fun.lg.g,n> ( <g> , <n> ) | <fun.lg.g.ln> ( <g> , <ln> ) | apply ( <fun.g.g,n> , <ln> , <g> )
<const.n> ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
<var.n> ::= depth | g.count.r | g.count.rw
<var.g> ::= g
<var.ln> ::= ln
<fun.n.g> ::= size | weight | weight.r | int
  <fun.n,n,n> ::= + | - | * | / | %
  <fun.n.ln> ::= length | max.index | min.index
  <fun.n.ln,n> ::= get
  <fun.n.lg> ::= length
  <fun.ln,n> ::= seq
  <fun.ln.n,n> ::= repeat
  <fun.g.g,n> ::= rotate.left | rotate.right | substring
  <fun.g.lg,n> ::= get
  <fun.lg.g,n> ::= split | repeat
  <fun.lg.g.ln> ::= split.w

```

- typed language: numbers $\langle n \rangle$, lists of numbers $\langle ln \rangle$, bit strings $\langle g \rangle$, list of bit strings $\langle lg \rangle$



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CHOOSE() and DIVIDE()

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    ⟨fun.n.ln,n⟩ (⟨ln⟩, ⟨n⟩) | ⟨fun.n.lg⟩ (⟨lg⟩)
  ⟨ln⟩ ::= ⟨var.ln⟩ | ⟨fun.ln.n⟩ (⟨n⟩) | ⟨fun.ln.n,n⟩ (⟨n⟩, ⟨n⟩) | apply (⟨fun.n.g⟩, ⟨lg⟩)
  ⟨g⟩ ::= ⟨var.g⟩ | ⟨fun.g.g,n⟩ (⟨g⟩, ⟨n⟩) | ⟨fun.g.lg,n⟩ (⟨lg⟩, ⟨n⟩)
  ⟨lg⟩ ::= ⟨fun.lg.g,n⟩ (⟨g⟩, ⟨n⟩) | ⟨fun.lg.g,ln⟩ (⟨g⟩, ⟨ln⟩) | apply (⟨fun.g.g,n⟩, ⟨ln⟩, ⟨g⟩)
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- typed language: numbers $\langle n \rangle$, lists of numbers $\langle ln \rangle$, bit strings $\langle g \rangle$, list of bit strings $\langle lg \rangle$
- MAP() as a pair of CHOOSE() ($\langle n \rangle$) and DIVIDE() ($\langle lg \rangle$)
- relevant (protected) functions: size, rotate.left, max.index, ...



Expressiveness?

Can express existing human-designed representations!

- standard GE⁴
 - CHOOSE() = int(substring(rotate.left(g, *(gl.count.rw, 8)), 8))
 - DIVIDE() = repeat(g, length(ln))
- HGE (Hierarchical GE)
 - CHOOSE() = max.index(apply(weight.r, split(g, length(ln))))
 - DIVIDE() = split(g, ln)
- WHGE (Weighted HGE)
 - CHOOSE() = max.index(apply(weight.r, split(g, length(ln))))
 - DIVIDE() = split.w(g, ln)

⁴an optimized version with null-invalidity

Bonus

The representation of grammar-based representations is grammar-based!
(*meta-GE*)

We can use any grammar-based EA for evolving these representations!



Bonus

The representation of grammar-based representations is grammar-based!
(*meta-GE*)

We can use any grammar-based EA for evolving these representations!

- we chose CFGGP (thought to be more efficient)
- with a diversity promotion mechanism



Fitness function

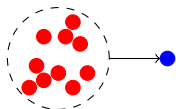
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Fitness function

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Redundancy (R)

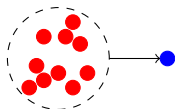


“Known” to be important: the lower, the better

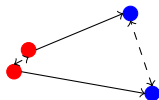
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Problem: evolving a representation **with good properties** for GE

Redundancy (R)



Non-locality (NL)

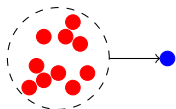


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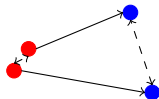
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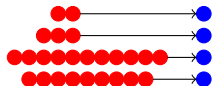
Redundancy (R)



Non-locality (NL)



Non-uniformity (NU)

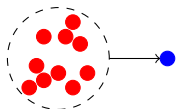


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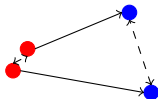
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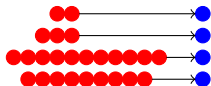
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Three variants:

- R (single-objective)
- R/NL (multi-objective)
- R/NL/NU (multi-objective)

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Goals – research questions

- RQ1 Can we evolve a representation which is better than the existing ones in terms of redundancy, locality, and uniformity?
- RQ2 Are the evolved representations also effective when used inside an actual EA?

Procedure

- 1 Learning (tot. 30 runs)
 - fitness (properties R, NL, NU) computed on a symbolic regression CFG (Pagie1) with short genotypes (256 bit)
- 2 Validation
 - properties R, NL, NU computed on 3 CFGs (Pagie1, K-Landscape, Text) with longer genotypes (1024 bit)
 - search effectiveness on the 3 problems (tot. 2250 runs)
 - comparison against human-designed representations (GE, HGE, WHGE)

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Answers to RQ1



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Answers to RQ1 and RQ2



RQ1: properties

RQ1 Can we evolve a representation which is better than the existing ones in terms of redundancy, locality, and uniformity?

	Search variant	Redundancy	Non-locality	Non-uniformity
Evolved	R	0.266	0.291	0.284
	R/NL	0.247	0.28	0.292
	R/NL/NU	0.261	0.29	0.288
Human	GE	0.990	1.000	0.000
	HGE	0.620	0.403	2.211
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- yes, we can!
- actual driving properties seem not to matter



RQ2: search effectiveness

RQ2 Are the evolved representations also effective when used inside an actual EA?

Search variant	Final best fitness			
	Best	Mean	Worst	
KLand-5	R	0.11	0.6	0.81
	R/NL	0.58	0.66	1
	R/NL/NU	0.55	0.7	1
	GE	1		
	HGE	0.58		
	WHGE	0.6		
Pagie1	R	3.42	338.66	4488.27
	R/NL	3.32	114.39	1142.28
	R/NL/NU	7.42	45.61	169.16
	GE	20.99		
	HGE	4.32		
	WHGE	2.75		
Text	R	6.5	65.12	176
	R/NL	7	88.06	176
	R/NL/NU	8.33	75.95	176
	GE	9.2		
	HGE	5.4		
	WHGE	5.4		

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	WHGE	5.4		

- best human-designed are better than the average evolved ones
- some evolved are better than the human-designed, on some problems



RQ2: search effectiveness

RQ2 Are the evolved representations also effective when used inside an actual EA?

	Search variant	Final best fitness		
		Best	Mean	Worst
KLand-5	R	0.11	0.6	0.81
	R/NL	0.58	0.66	1
	R/NL/NU	0.55	0.7	1
	GE	1		
	HGE	0.58		
	WHGE	0.6		
Pagie1	R	3.42	338.66	4488.27
	R/NL	3.32	114.39	1142.28
	R/NL/NU	7.42	45.61	169.16
	GE	20.99		
	HGE	4.32		
	WHGE	2.75		
Text	R	6.5	65.12	176
	R/NL	7	88.06	176
	R/NL/NU	8.33	75.95	176
	GE	9.2		
	HGE	5.4		
	WHGE	5.4		

- best human-designed are better than the average evolved ones
- some evolved are better than the human-designed, on some problems
- representations evolved with R only look better: redundancy is important
 - consistent with literature



Summarizing

Key findings:

- automatic design of representations of realistic complexity is feasible
- good properties can be achieved!
- ... \nrightarrow good search effectiveness
 - shed some light on relevance of representation properties

Open issues:

- are R, NL, NU the right properties?
- which part of the property is “in the CFG”?



Thanks!